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Arched Vaults Made of Corrugated Sheets: Natural Vibration Frequencies

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Abstract. *According to the recent Russian norms, when designing building structures, it is necessary to conduct a dynamic analysis of wind loads, which previously was required not in all cases. In arched vaults of profiled arched self-supporting flooring, it was not always necessary to determine the frequencies and forms of natural vibrations. These parameters can be established using the finite element method. Taking into consideration the complex and lengthy modeling process of arched vaults, in particular the contact areas in regular transverse joints, a large number of finite elements in the models and, as a consequence, considerable time for their calculation, it was necessary to identify a sufficient level of detailing of a finite element model for correct calculations of frequencies and forms of vibrations considered in the operation of structures. The influence of the detailing of the finite element model of arched vaults made of profiled flooring on the determination of their natural frequency is revealed. To substantiate the parameters of the finite element model, it was studied how the results of the calculation was influenced by edge effects, the presence of friction in the joints of corrugations of the profiled flooring, the influence of rubber gaskets in the joints on the work of the arched vault. Much attention is paid to the features of the design scheme associated with the choice of the number of sheets of profiled flooring, taking into account the contact nodes, friction forces, etc. It is established that the first vibration frequency is little dependent on the number of sheets of corrugated flooring and the presence of rubber gaskets in the joints. For subsequent frequencies, the difference can be significant.*
Key words*: finite element method, finite element models, frequencies and forms of natural oscillations of arched vaults, median surfaces, plates, border conditions.*

АРОЧНЫЕ СВОДЫ ИЗ ПРОФИЛИРОВАННОГО НАСТИЛА: ЧАСТОТЫ СОБСТВЕННЫХ КОЛЕБАНИЙ

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Аннотация. *Согласно современным требованиям российских норм при проектировании строительных конструкций необходимо выполнять динамический анализ ветровых нагрузок, что ранее требовалось не во всех случаях. В арочных сводах из профилированного арочного самонесущего настила не всегда было необходимо определять частоты и формы собственных колебаний. Данные параметры можно установить с использованием метода конечных элементов. Принимая во внимание сложный и длительный процесс моделирования арочных сводов, в частности зон контакта в регулярных поперечных стыках, большое количество конечных элементов в моделях и, как следствие, значительное время для их расчета, потребовалось выявить достаточную степень детализации конечно-элементной модели для корректного вычисления частот и формы колебаний рассматриваемых в работе конструкций. Выявлено влияние детализации конечно-элементной модели арочных сводов из профилированного настила на определение их собственной частоты. Для обоснования параметров конечно-элементной модели изучено как на результаты расчета влияют краевые эффекты, наличие трения в стыках гофр профилированного настила, влияние на работу арочного свода резиновых прокладок в стыках. Уделено большое внимание особенностям расчетной схемы, связанным с выбором числа листов профилированного настила, учетом контактирующих узлов, силы трения и др. Установлено, что первая частота колебаний мало зависит от числа листов профнастила и наличия в стыках резиновых прокладок. Для последующих частот разница может быть значительной.*
Ключевые слова*: метод конечных элементов, конечно-элементные модели, частоты и формы собственных колебаний арочных сводов, серединные поверхности, пластины, граничные условия.*

Introduction

It is known, that any real constructive system is oscillating and its dynamic behavior under conditions of variable external loading can be estimated by comparing the dominant frequencies of external forces and the natural frequencies of the systems.

Modern structures of buildings are complex. They consist of many different constractions and systems of connections. The components of the system interact with each other and their totality gener-

Fig. 1. *An example of the use of arch system covering a sport building*

Рис. 1. *Пример использования арочного свода в спортивном сооружении*

Fig. 2. *Arch system – covering a shopping center*

Рис. 2. *Арочный свод в покрытии торгового центра*

ates different natural frequencies and natural vibrations. The theoretical determination of natural vibrations and natural frequencies is a complex dynamic problem. In the practice of designing products for mechanical engineering, the methods for determining the natural frequencies of typical parts and structural elements are well developed [1–3].

In former time in Russia for the design of buildings and structures it was commonly not necessary to determine the natural frequencies, only in some cases specified by the standards. The experimental and theoretical foundations for calculating the interaction of buildings and structures with a dynamic wind load were developed in the late 1950's and early 1960's. Since that time the rules are practically not changed. Such calculations were labor-intensive, required a large amount of computer calculations. Furthermore the capabilities of the computers in this time were limited, they did not allow to apply developed calculation methods [4]. Until recently according to Russian standards (SNiP 2.01.07-85 * "Loads and actions") the dynamic component of wind load could be ignored, if the following three conditions were fulfilled: the height of building is maximum 40 m in case of a multiy-story building; the height of building 36 m in case of a single-story industrial building with the ratio height to width is less than 1.5; the building is situated in an area that is surrounded by obstacles of 10 m height or more.

It was assumed, that in the mentioned cases the natural frequencies of buildings built from traditional materials were significantly higher than the forced frequencies of dynamic wind load [5], which minimize the risk of resonance.

Now, in accordance with the changed standards for the design of building elements (SP 20.13330.2016. Updated version of SNiP 2.01.07-85 * "Loads and actions"), the method of calculation of dynamic wind loads acting on buildings has been changed. According to the modern standards the calculations of all structures must be carried out taking into account the dynamic effects.

The tasks of such calculations are to determine the natural frequencies of the projected objects in order to exclude the appearance of resonance. In cases if resonance appears, it is necessary to determine the maximum stresses in structural elements that arise during the oscillation process and compare it with the maximum permissible values determined from the strength and rigidity conditions [5].

In connection with the foregoing, one of the most important questions is the determination the optimal parameters of different beam systems and plate systems [6–9].

At the present, arch systems made of trapezoidal sheets are widely used in the construction industry. They are made of thin-walled trapezoidal sheets bent to a cylindrical shape and are used as a roof for buildings and structures [10–13].

The main types of loads acting on the arch system are atmospheric – snow and wind loads [14]. Under their influence, a complex stress-strain state arises in the elements, which is a result of the three-dimensional characteristic of the arch systems made of profiled sheet. Such systems are of low stiffness, and the natural frequencies of these structures can be close to the forced frequency of dynamic wind load, including vortex induced vibrations and gust-inducted vibrations. Forced excitations can also be caused by seismic loads or other base excitations. Therefore the designs of such buildings and structures should be done by considering the dynamic behavior.

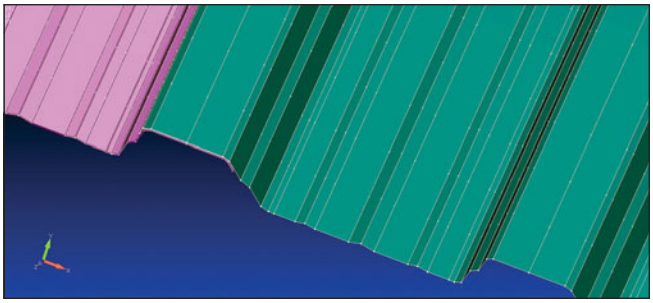


Fig. 3. *The places of transverse joints with a double sheet at the upper flanges*

Рис. 3. *Поперечные стыки с двойной толщиной на верхних полках*

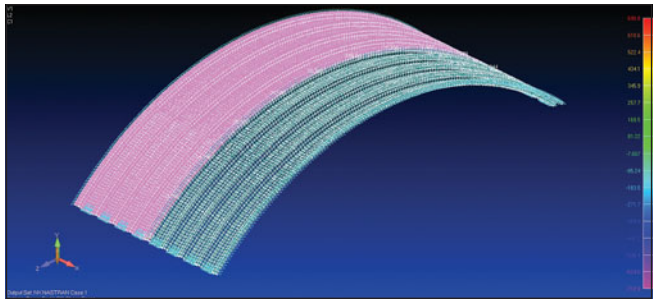


Fig. 4. *FE-model of an arch system made of two overlapping workpieces (profile AH60, 1 mm thickness)*

Рис. 4. *Конечно-элементная модель арочного свода, выполненная из двух арочных заготовок (профиль AH60, толщина – 1 мм)*

Formulation of the problem

As an object of investigation, an arch system made of trapezoidal corrugated sheet was chosen. Such designs have recently found wide application. Examples of structures considered in this publication are shown in *Figures 1* and *2*.

The structural feature of arch systems made of trapezoidal sheet is the presence of transverse joints *(Figure 3)*.

The technologically possible width of a trapezoidal profiled and arched sheet varies from 500 to 900 mm depending on the height (type) of the profile. Transverse joints in arch systems is a result in periodic sections with a double thickness of the upper flange of profile, which leads to an increased rigidity of shell (see *Fig. 3*). This must be taken into account in the calculations of these constructions, including dynamic calculations.

Theory. General analytical methods for determining the natural frequencies of mechanical systems are difficult to use for the object of study.

Recently, scientists have attempted to improve the theory of determining the natural frequencies of different design systems [15–17], including using finite-element models. But nevertheless, at the present time there is no universal method for solving this problem.

In accordance with modern design Russian standards (SP 20.13330.2016) of all constructions, it is necessary to perform dynamic calculations. It includes the determination of the natural frequencies and natural vibrations and the consideration of the dynamic component of wind load. The characteristic value of the main wind load in accordance with Russian standards (SP 20.13330.2016) is defined as the sum of the average component w_m and the dynamic component w_p:

$$w = w_m + w_p. \qquad (1)$$

The average component of the main wind load depends on the height of the building, the wind area and the type of terrain. It does not depend on the natural frequency.

The characteristic value of the dynamic component of the main wind load at the equivalent height must be determined in accordance with Russian standards as follows:

- for structures and their structural elements, in which the first natural frequency, f_1, Hz, is greater than the limit of the natural frequency f_{lim}, the dynamic component of wind load can be determined according to the formula

$$w_p = w_m \xi(z_e) \nu, \qquad (2)$$

where w_m is the average value of the component of the main wind load; ν is the coefficient of three-dimensional correlation of dynamic wind load; $\xi(z_e)$ is the wind pressure pulsation coefficient;

- for all structures and structural elements in which $f_1 < f_{\text{lim}} < f_2$ the characteristic value of the dynamic component of the main wind load must be determined from SP 20.13330.2016:

$$w_p = w_m \xi \xi(z_e) \nu, \qquad (3)$$

where f_2 is the second natural frequency; ξ is the coefficient of dynamics, determined depending on the logarithmic decrement of the oscillations δ and the parameter of first natural frequency f_1:

$$\varepsilon_1 = \frac{\sqrt{w_0 k(z_{эк}) \gamma_f}}{940 f_1}, \qquad (4)$$

where w_0, Pa, is the characteristic value of wind pressure; $k(z_{эк})$ is a coefficient, taking into account the change in wind pressure for altitude $z_{эк}$; γ_f is a safety factor.

Therefore, for a correct design of arch systems it is necessary to know at least the first two natural frequencies. In the Russian standards the method for determining the natural frequencies is not regulated. This is the purpose of research.

Methods. For research, the finite element method (FEM) was used. Finite-element models make it possible to take into account all structural features of arch systems, such as the geometry of the cross section of arched blanks, the presence of transverse joints and connecting elements.

Taking into account that the

arch system of this publication have a transverse corrugation profile (are geometrically orthotropic), have regular technological joints (technologically orthotropic), theoretical the determination of the natural frequencies is not possible. The natural frequencies and natural vibrations of arch system were determined using the FEM. Finite-element models make it possible to take into account all the above mentioned features of the constructions under dynamic load.

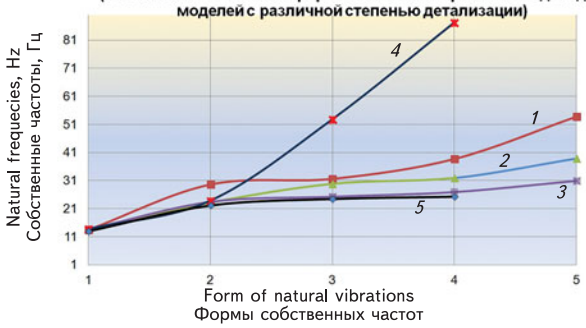
The objectives of this research

Considering the large number of input parameters, the long and complicated process of arch simulation and the considerable time of calculating the finite-element models, it became necessary to solve the problem of the required degree of detailing of finite-element model.

When choosing the degree of detail of the finite-element model of arch arches, it is necessary to keep in mind the complex and lengthy modeling process, a large number of finite elements and the long time for their calculation.

In order to obtain correct results for the determination of natural frequencies and natural vibrations of arch system, it was required to investigate the following things:

- which part of the arch system should be modeled (with what number of profiled sheets) to exclude the edge effect on the results of determining the natural frequencies of vibration;
- it is necessary or not necessary to model the transverse joints (see *Fig. 3*), taking into account friction in them, possible slippage and the presence of connecting elements;
- whether it is necessary to model rubber gaskets for connecting elements in transverse joints.



1 — one profiled sheet; 2 — two profiled sheets with contact surfaces in the transverse joints; 3 — four profiled sheets, with contact surfaces in the transverse joints; 4 — monolith, four profiled sheets without contact; 5 — four profiled sheets, taking into account the contact of surfaces and rubber gaskets
1 — один профилированный лист; 2 — два профилированных листа с учетом контакта поверхностей в поперечных стыках; 3 — четыре профилированных листа с учетом контакта поверхностей в поперечных стыках; 4 — монолитный вариант, четыре профилированных листа без поперечных стыков и без учета контакта в них; 5 — четыре профилированных листа с учетом контакта поверхностей и резиновыми вкладышами

Fig. 5. ***Results of determination of natural frequencies of the arch systems***

Рис. 5. ***Результаты определения собственных частот арочных сводов***

To solve the problem, a numerical experiment was carried out. A number of geometric and discrete models of various variants of arch system consisting of one, two, four and ten workpieces with transverse joints are considered. A variant of the arch system of four workpieces with rubber gaskets in the transverse joint in the places of connecting elements is considered, and also the same set of ten blanks without regard to joints ("monolithic"). To analyze the obtained results of determining the natural frequencies and natural vibration, all finite-element models of arch systems were made with the same span, rise of arch, thickness and type of profile.

Geometric and discrete models of arch systems, external data, boundary conditions and visualization and processing of analysis results were carried out using PRE / POST processor FEMAP. To determine the natural frequencies and natural vibrations, the universal block of the "NX NASTRAN" software package "Normal Modes" is used.

An example of a finite-element model of an arch system made of two workpieces from the AH60 profile, a span of 4 m, a rise of arch of 0.8 m, is shown in *Figure 4*.

In finite-element models of arch systems, lamellar finite elements of the QUAD-4 type were used, the calculations were carried out taking into account the physical non-linearity of the material of the arched.

Particular attention was paid to the contact surfaces of the transverse joint while creating the finite-element models. They do not have boundary conditions for kinematic fixing.

The special feature of the created FEM were modeling the conditions of contact conjugation line of shell elements and taking into account the changes of these conditions under dynamic loading.

For a given boundary contact problem, contact zones of a variable configuration are characteristic [18].

When determining the natural frequencies of arch system, it was taken into account that the coordinates of the nodes of the contacting surfaces of the upper workpiece should always be larger than the lower coordinates. Physically it means the exclusion of interpenetration of surfaces of the contacting arched workpieces. In the finite-element models for modelling the contact zones the surface-to-surface type of contact elements

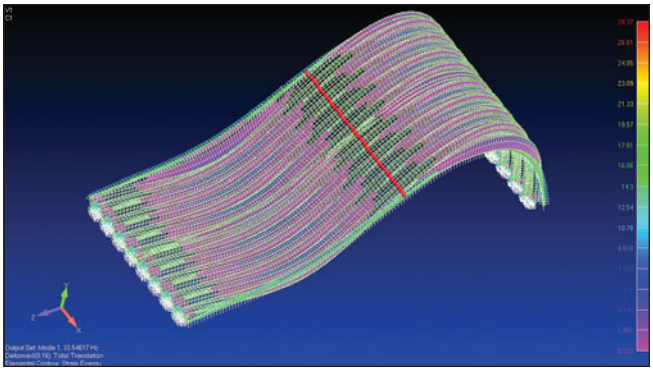


Fig. 6. *The first form of natural vibration (longitudinal)*

Рис. 6. *Первая форма собственных колебаний (продольные)*

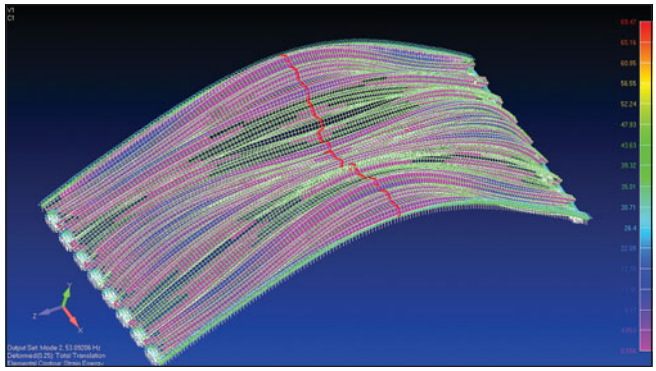


Fig. 7. *The second form of natural vibration (transverse)*

Рис. 7. *Вторая форма собственных колебаний (поперечные)*

was used. Following conditions were taken into account:

• in finite-element models of arch systems made of two or more profiled sheets, the distance between the middle surfaces of the contact elements was assumed to be equal to the thickness of the profile;

• the contacting nodes and the final elements of arch system are not known in advance;

• friction between sheets is modeled in accordance with the law of Coulomb friction in the classical formulation;

• the material is isotropic and obeys Hooke's law;

• deformations and angles of rotation are small;

• the appearance and disappearance of contact zones (pairs of nodes, joints) can have any sequence;

• the adhesion and slippage of sheets is simulated.

Results and discussion

The Lanczos complex method was used. The natural frequencies and natural vibrations were determined, with consideration of energy dissipation, including the version of FE-model with rubber gaskets.

Some the results of calculations are:

a – one profiled sheet:
f_1 = 13.38 Hz; f_2 = 29.53 Hz; f_3 = 31.56 Hz; f_4 = 38.62 Hz; f_5 = 53.67;

b – two profiled sheets with contact of surfaces:
f_1 = 13.52 Hz; f_2 = 23.12 Hz; f_3 = 29.83Hz; f_4 = 31.93 Hz; f_5 = 38.66 Hz;

c – four profiled sheets, with contact surfaces:
f_1 = 13.59 Hz; f_2 = 23.41 Hz; f_3 = 25.24 Hz; f_4 = 26.90 Hz; f_5 = 30.91 Hz.

The found frequencies of the lower vibrational modes of of the arch systems are shown in *Figure 5.*

Figure 6 shows the first form of natural vibration of the arch system of the model with four profiled sheets; frequency = 13.59 Hz.

In *Figure 7* – the second form of natural vibration of the arch system of four profiled sheets, frequency = 23.41 Hz.

Conclusions

To study the influence of segment length of arch roof the models with one, two and four profiled sheet with transverse joints, but without rubber gaskets in joints can be compared. It was found that the effect of segment length on the first natural frequency is very low with maximum 2 % deviation. At higher natural frequencies, however, significant differences occur. The deviation increases with the height of the natural frequency, in the 5th natural frequency, a deviation of up to 74 % were found. The edge effect of the FE-model plays an increasing role with the increasing order number of the desired natural frequency.

1. Using a finite element analysis package NATRAN, it is possible to determine the frequencies of natural oscillations with sufficient accuracy for engineering calculations.

2. Minimum four profiled sheets of the arch system should be modeled (to exclude the edge effect on the results of determining the natural frequencies of vibration;

3. The effect of transverse joints becomes apparent when models 3 and 4 of *Fig. 5* are compared. At the 1st and 2nd natural frequency, which play a special role in most dynamic calculation methods, the influence of transverse joints with a maximum of 2 % is negligible. However, for the determination of higher natural frequencies, the influence of transverse joints should be taken into account.

4. The influence of rubber gaskets in longitudinal joints becomes apparent by comparing model 3 and 5 of *Fig. 5*. The consideration of the rubber gaskets generally leads to a reduction of the natural frequency. The deviation can be up to 7 %. It can be caused partly due to the increased structural damping, but also due to less rigidity of system.

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